Interpolation of Order Statistics in Stable Distribution

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\textbf{Keywords.} Best linear unbiased interpolation or prediction; Projection; Hilbert space; Order statistics.

1 Introduction

In this paper we derive the Best Linear Unbiased Interpolation for the missing order statistics from a stable distribution using the well-known projection theorem. The proposed interpolation method only needs the first two moments of both sides of a missing order statistic. A simulation study is performed to compare the proposed method with a few interpolation methods for some stable distributions.

2 Best linear interpolation

All results in this section will be stated for a Hilbert space $L^2 = \{X, \ E(X^2) < \infty\}$ with the inner product $<X_1, X_2> = E(X_1X_2)$.

Let $X_1, \ldots, X_n$ be a sequence of random variables with common pdf $f(x)$ and cdf $F(x)$ and let $Y_1 \leq \cdots \leq Y_n$ be the corresponding order statistics. Using the projection theorem, the conditional expectation $E_{\mathcal{M}(X_1,\ldots,X_n)}(X)$ is the best mean square predictor of $X$ in $\mathcal{M}(X_1,\ldots,X_n)$, see Brockwell and Davis [1991].

\textbf{Lemma 2.1} (Best order statistics interpolator). Let $L^2$ be a Hilbert space and $P_{\mathcal{M}}$ denote the projection mapping onto a closed subspace $\mathcal{M}$. If $Y_1 \leq \cdots \leq Y_r \leq Y_s \leq \cdots \leq Y_n$ is order statistics of a distribution, then

$$P_{\mathcal{M}(Y_1,\ldots,Y_r,Y_s,\ldots,Y_n)}(Y_l) = P_{\mathcal{M}(Y_r,Y_s)}(Y_l),$$

(1)
where $r < l < s$.

**Lemma 2.2** Best linear interpolator of $l$'th order statistic ($r < l < s$) can be obtained as follows

$$P_{\hat{y}(Y_r, Y_s)}(Y_l) = a_0 + a_1 Y_r + a_2 Y_s,$$

where

$$a_1 = \frac{\rho_{r, l} - \rho_{r, s} \rho_{l, s} \sigma_{Y_i}}{(1 - \rho_{r, s}^2) \sigma_{Y_r}}, \quad a_2 = \frac{\rho_{l, s} - \rho_{r, s} \rho_{r, l} \sigma_{Y_i}}{(1 - \rho_{r, s}^2) \sigma_{Y_s}}, \quad a_0 = \mu_{Y_i} - a_1 \mu_{Y_r} - a_2 \mu_{Y_s},$$

and $\mu_{Y_i} = \mathbb{E}(Y_i)$, $\sigma_{Y_i}^2 = \text{var}(Y_i)$ and $\rho_{i, j} = \text{corr}(Y_i, Y_j)$, $i, j = r, l, s$.

**Corollary 2.1** Let $\hat{Y}_{i, \text{BLUI}}$ stands for the best linear unbiased interpolation of $Y_i$. Under a few conditions, the BLUI is reduce to

$$\hat{Y}_{i, \text{BLUI}} = \begin{cases} \mu_{Y_i} + \rho_{r, l} \frac{\sigma_{Y_i}}{\sigma_{Y_r}} (Y_r - \mu_{Y_r}), & \text{if } |l - s| \text{ is large,} \\ \mu_{Y_i} + \rho_{l, s} \frac{\sigma_{Y_i}}{\sigma_{Y_s}} (Y_s - \mu_{Y_s}), & \text{if } |r - l| \text{ is large,} \\ \mu_{Y_i} + \rho_{r, l} \frac{\sigma_{Y_i}}{\sigma_{Y_r}} (Y_r - \mu_{Y_r}) + \rho_{l, s} \frac{\sigma_{Y_i}}{\sigma_{Y_s}} (Y_s - \mu_{Y_s}), & \text{if } |r - s| \text{ is large,} \\ \mu_{Y_i}, & \text{if } |l - r| \text{ and } |l - s| \text{ are large.} \end{cases}$$

**References**