Interpolation of Order Statistics in Stable Distribution

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1 Introduction

In this paper we derive the Best Linear Unbiased Interpolation for the missing order statistics from a stable distribution using the well-known *projection theorem.* The proposed interpolation method only needs the first two moments of both sides of a missing order statistic. A simulation study is performed to compare the proposed method with a few interpolation methods for some stable distributions.

2 Best linear interpolation

All results in this section will be stated for a Hilbert space $L^2 = \{X, \ \ E(X^2) < \infty\}$ with the inner product $< X_1, X_2 >= E(X_1 X_2)$.

Let $X_1, \ldots, X_n$ be a sequence of random variables with common pdf $f(x)$ and cdf $F(x)$ and let $Y_1 \leq \cdots \leq Y_n$ be the corresponding order statistics. Using the projection theorem, the conditional expectation $E_{\mathcal{M}(X_1,\ldots,X_n)}(X)$ is the best mean square predictor of X in $\mathcal{M}(X_1, \ldots, X_n)$, see Brockwell and Davis [1991].
Lemma 2.1 (Best order statistics interpolator). Let L^2 be a Hilbert space and $P_{\mathcal{M}}$ denote the projection mapping onto a closed subspace $\mathcal{M}$.
If $Y_1 \leq \cdots \leq Y_r \leq Y_s \leq \cdots \leq Y_n$ is order statistics of a distribution, then

$$P_{\mathcal{M}(Y_1,\ldots,Y_r,Y_s,\ldots,Y_n)}(Y_l) = P_{\mathcal{M}(Y_r,Y_s)}(Y_l), \tag{1}$$

where $r < l < s$.

Lemma 2.2 Best linear interpolator of l'th order statistic ($r < l < s$) can be obtained as follows

$$P_{\bar{sp}\{1,Y_r,Y_s\}}(Y_l) = a_0 + a_1 Y_r + a_2 Y_s, \tag{2}$$

where

$$a_1 = \frac{\rho_{r,l} - \rho_{r,s}\rho_{l,s}}{(1-\rho_{r,s}^2)}\frac{\sigma_{Y_l}}{\sigma_{Y_r}}, \quad a_2 = \frac{\rho_{l,s} - \rho_{r,s}\rho_{r,l}}{(1-\rho_{r,s}^2)}\frac{\sigma_{Y_l}}{\sigma_{Y_s}}, \quad a_0 = \mu_{Y_l} - a_1\mu_{Y_r} - a_2\mu_{Y_s}, \tag{3}$$

and $\mu_{Y_i} = E(Y_i)$, $\sigma_{Y_i}^2 = \mathrm{var}(Y_i)$ and $\rho_{i,j} = \mathrm{corr}(Y_i, Y_j), i, j = r, l, s$.

Corollary 2.1 Let $\hat{Y}_{l,\mathrm{BLUI}}$ stands for the best linear unbiased interpolation of Y_l. Under a few conditions, the BLUI is reduce to

$$\hat{Y}_{l,\mathrm{BLUI}} = \begin{cases} \mu_{Y_l} & + \quad \rho_{r,l}\frac{\sigma_{Y_l}}{\sigma_{Y_r}}(Y_r - \mu_{Y_r}), & \text{if} \quad |l-s| \text{ is large,} \\ \mu_{Y_l} & + \quad \rho_{l,s}\frac{\sigma_{Y_l}}{\sigma_{Y_s}}(Y_s - \mu_{Y_s}), & \text{if} \quad |r-l| \text{ is large,} \\ \mu_{Y_l} & + \quad \rho_{r,l}\frac{\sigma_{Y_l}}{\sigma_{Y_r}}(Y_r - \mu_{Y_r}) + \rho_{l,s}\frac{\sigma_{Y_l}}{\sigma_{Y_s}}(Y_s - \mu_{Y_s}), & \text{if} \quad |r-s| \text{ is large,} \\ \mu_{Y_l}, & & \text{if} \quad |l-r| \text{ and } |l-s| \text{ are large.} \end{cases}$$

References

Brockwell, P. J., Davis, R. A. (1991). Time series: theory and methods, second ed., Springer Series in Statistics, Springer-Verlag, New York.