Robust estimation in single index models with asymmetric errors

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Parametric and nonparametric models are two important branches of of regression analysis. An alternative between them is given by semiparametric models, that combine parametric components with nonparametric ones, retaining the advantages of both types of models and avoiding their drawbacks.

A relevant topic in this broad class of models is given by Simple Index Models (SIM) in which the response variable y is related to the covariate vector x through the equation

$$y = \eta(\beta' x) + \epsilon\,, \tag{1}$$

where $\beta, x \in R^q$ and $\eta : R \to R$ is a univariate real valued function. For the sake of identifiability, it is assumed with no loss of generality that $\|\beta\| = 1$ and the first component of β is positive, where $\|\cdot\|$ denotes the Euclidean norm.

Theses models reduce the dimensionality of the covariates by means of the single index $\beta' x$, capturing at the same time a possible nonlinear trend by means of the function η. In this way these models cope with the *curse of dimensionality*. They can also be seen as a technique of dimension reduction since, if β can be estimated in an efficient way, variable $\beta' x$ can be use as a carrier to estimate nonparametrically the function η.

Most of the literature assumes that the errors distribution has finite second moment and mean zero. However, in the robust framework this assumption is generally replaced by the symmetry of the errors term distribution, in order to achieve

Fisher–consistent estimators. In some situations the practitioner faces the problem of asymmetric errors, as it is the case when the error term distribution belongs to a class of exponential families, for instance the log–gamma distribution. We focus on the problem of robust estimating the parametric and nonparametric components of model (**??**) when the density of the error ϵ is of the form

$$g(\epsilon, \alpha) = Q(\alpha) \exp^{\alpha t(\epsilon)},$$

with $\alpha > 0$ a nuisance parameter and $t : R \to R$ a continuous function with unique mode at ϵ_0, which includes the Gamma distribution with a log link.

A family of robust estimators for η and β based on a three–step procedure related to a profile approach is proposed. When nuisance parameters are present, they may be estimated using a preliminary S-estimator which will allow to define also the tuning constant. In particular, we have introduced a robust consistent estimator of the nuisance parameter for the usual regression model with symmetric errors and for the log–gamma regression model.

Consistency results for the robust profile proposal is obtained. A preliminary simulation study is presented so as to validate the good behaviour of the estimators under the central model and under different contaminated scenarios.