Smooth Plus Rough Variation of Random Functions: the Interplay Between Rank, Resolution, and Scale

M-H. Descary* and V.M. Panaretos

Section de Mathématiques, Ecole Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland; marie-helene.descary@epfl.ch, victor.panaretos@epfl.ch

Keywords. *Covariance operator; Functional Data Analysis; Functional PCA; Matrix completion.*

Functional data analyses typically proceed by smoothing, followed by functional PCA. This paradigm implicitly assumes that any roughness is due to nuisance noise. Nevertheless, relevant functional features such as time-localised or short scale variations may indeed be rough. These will be confounded with the smooth components of variation by the smoothing/PCA steps, potentially distorting the parsimony and interpretability of the analysis. We will explore how both smooth and rough variations can be recovered on the basis of discretely observed functional data. Assuming that a functional datum arises as the sum of two uncorrelated components, one smooth and one rough, we develop identifiability conditions for the estimation of the two corresponding covariance operators. We construct nonlinear estimators of the smooth and rough covariance operators and their spectra via matrix completion, and establish their consistency and rates of convergence. We then use them to recover the smooth and rough components of each functional datum, effectively producing separate functional PCAs for smooth and rough variation.

References

Descary, M-H. & Panaretos, V.M. (2015). Smooth plus rough variation of random functions: the interplay between rank, resolution, and scale. *Submitted.*