Robust Wald-Type Tests under Random Censoring

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Randomly censored survival data are frequently encountered in several applied sciences including biomedical sciences and reliability applications; analyzing these data are very important to obtain inferences for all these applications. The underlying censoring mechanism may or may not be known and several semi-parametric approaches are available in the literature for the analysis of such data. However, these semi-parametric procedures are not as efficient as the classical maximum likelihood procedures which can be implemented under some parametric assumptions of the distribution and random censoring scheme. However, as in the case of several other types of data, the maximum likelihood methods for censored data are also highly non-robust with respect to the outlying observations. Since it is common to have some potential outliers in several real-life applications with survival data, either due to erroneous input or some unknown underlying mechanism (like some data points might come from a different group), suitable robust procedures having good efficiency are very useful in practice.

Under the context of survival data with random censoring, there are very few approaches of robust estimation that consider the fully parametric set-up to gain more efficiency; Wang (1999) and Basu et al. (2006) proposed two such estimation approaches. The first one develops the general theory of M-estimation under the set-up of randomly censored data while the second one considers a particular M-estimator based on the density power divergence (DPD; Basu et al., 1998) exhibiting highly efficient and robust performances. Recently, Ghosh and Basu (2014) have extended these approaches of M-estimation and the minimum DPD estimation for the cases where we have some stochastic covariates along with the randomly censored response. However, there is no literature on the robust test of hypothesis under the

fully parametric set-up with censored data, although this is very important for any real-life inference problems.

In this paper, we propose the Wald-type test for testing common statistical hypothesis for randomly censored data. We consider both the simple and composite hypotheses and use the minimum DPD estimators under fully parametric set-up; the high efficiency of the estimator used yield high power for our testing procedure. We have also derived the general theory and properties of our proposed Wald-type test statistics for the general M-estimators and demonstrated the usefulness of its particular member, the Minimum DPD estimator based test statistics, through appropriate theoretical and numerical illustrations. In this way, we have proposed a consistent estimator of the asymptotic variance of the M-estimator based on the sample data without any assumption on the form of censoring scheme. We have also presented the robustness of our proposal theoretically through suitable influence function analysis and numerically through appropriate simulations and real data examples. Finally, we have briefly indicated how to extend our proposed Wald-type test statistics for the two sample problem with random censoring and stochastic covariates.

References

Basu, S., Basu, A., and Jones, M. C. (2006). Robust and efficient parametric estimation for censored survival data. *Annals of the Institute of Statistical Mathematics*, **58(2)**, 341–355.

Basu, A., Harris, I. R., Hjort, N. L., and Jones, M. C. (1998). Robust and efficient estimation by minimising a density power divergence. *Biometrika*, **85(3)**, 549–559.

Ghosh, A. & Basu, A. (2014). Robust and Efficient Parameter Estimation based on Censored Data with Stochastic Covariates. *ArXiv Pre-print*, arXiv:1410.5170 [math.ST].

Wang, J. L. (1999). Asymptotic Properties of M-Estimators Based on Estimating Equations and Censored Data. *Scandinavian journal of statistics*, **26(2)**, 297–318.