1 Robust Tests for Exponential Distribution Data based on Repeated Median Estimator

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Summary. In this study, we use the robust Brown-Forsythe and robust Modified Brown-Forsythe test statistics proposed by [Karagöz & Saraçbaşı(2016)] for the one-way ANOVA under heteroscedasticity for the exponentially distributed data with outliers. In order to get these robust test statistics, repeated median robust estimator of the mean and variance of exponential distribution are obtained. We consider balanced and unbalanced sample sizes with homogeneous and heterogeneous variances. The simulations results show up that the proposed robust tests have a good performance.

Keywords: ANOVA, Exponential Distribution, Brown-Forsythe, Modified Brown-Forsythe, Repeated Median

1.1 Introduction

ANOVA is one of the most commonly used models in many fields such as medicine, engineering, psychology, sociology, etc. In general, the main interest of this paper is testing the homogeneity of group means of non-normal data. One-way ANOVA is based on assuming the normality of the observations and the homogeneity of group variances. The classical ANOVA uses the F-test statistic. If the assumptions of normality and homogeneity of variances are invalid and there are also outliers are present, classical ANOVA does not give accurate results. Therefore, test statistics based on robust methods should be used instead of the classical ANOVA.

The main theme of the paper is the estimation of the parameters λ in the exponential density function $f_\lambda(x) = \lambda \exp[-(\lambda x)]$, where $x, \lambda > 0$ and the parameter λ is called a rate parameter. The reciprocal $1/\lambda$ is known as the scale parameter.

The one-way classification of analysis of variance for non-normal data with heteroscedastic variance has been studied for a long time. In the case of disruption of assumptions, instead of ANOVA F-test, many test statistics have been developed such as Welch, Brown-Forsythe and Modified Brown-Forsythe. The difference of this study from the other studies are the RBF and $RMBF$ test statistics based on the repeated median estimator of mean and variance of the exponential distribution are considered. Robust Brown-Forsythe test statistics is given by [Karagöz & Saraçbaşı(2016)] as following

$$RBF = \frac{\sum_{i=1}^{k} n_i(\hat{\mu}_{ri.} - \hat{\mu}_{r..})^2}{\sum_{i=1}^{k}(1 - n_i/N)\hat{\sigma}_{ri}^2}. \tag{1.1}$$

RBF test statistic has F_{k-1,v_r} distribution with $k-1$ and v_r degrees of freedom. v_r is defined as $v_r = \frac{[\sum_{i=1}^{k}(1-n_i/N)\hat{\sigma}_{ri}^2]^2}{\sum_{i=1}^{k}(1-n_i/N)^2\hat{\sigma}_{ri}^4/(n_i-1)}$.

Robust modified Brown-Forsythe test statistics is given by [Karagöz & Saraçbaşı(2016)] as following

$$RMBF = \frac{\sum_{i=1}^{k} n_i(\hat{\mu}_{ri.} - \hat{\mu}_{r..})^2}{\sum_{i=1}^{k}(1 - n_i/N)\hat{\sigma}_{ri}^2} \tag{1.2}$$

$RMBF$ test statistic has $F_{v_{r_1},v_r}$ distribution with v_{r_1} and v_r degrees of freedom. The numerator degrees of freedom v_{r_1} is defined as

$$v_{r_1} = \frac{[\sum_{i=1}^{k}(1-n_i/N)\hat{\sigma}_{ri}^2]}{\sum_{i=1}^{k}\hat{\sigma}_{ri}^4 + [\frac{\sum_{i=1}^{k} n_i\hat{\sigma}_{ri}^2}{N}]^2 - 2\frac{\sum_{i=1}^{k} n_i\hat{\sigma}_{ri}^4}{N}}$$

In the above equations, $\hat{\mu}$ and $\hat{\sigma}$ are the robust repeated median estimators of mean and variance of exponential distribution.

1.2 Conclusion

In this study we work on the robust test statistics RBF and RGBF for the exponentially distributed data by using the robust repeated median estimators. In the simulation study, using different experimental designs, type I error risks of the robust test statistics for the exponential distribution are obtained for k=4,8 groups. We consider balanced and unbalanced sample sizes with homogeneous and heterogeneous variances. The simulations results show up that the proposed robust tests have a good performance.

References

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