Performance analysis and robustification of sequential statistical decision rules

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1 Introduction

Problems of statistical decision rules construction appear in many fields of real life. With sequential approach (**?**) it is possible to construct optimal decision rules w.r.t. the expected number of observations (sample size) provided the requested accuracy level (given small values for error probabilities upper bounds) is satisfied (**?**). This feature of sequential statistical decision rules plays an important role for practical use, especially in quality control, medicine, risk analysis.

In practice, the hypothetical model is often distorted (**?**), (**?**): observations contain "outliers", prior probability distributions are not true, the data can be incomplete. For these situations the mentioned optimal property of sequential decision rules is not valid (**?**), (**?**), and in some situations the traditionally used sequential decision rules are even not defined.

As a result, the problem of sequential decision rules performance analysis under distortions of different types is important (**?**). This problem should be considered together with the problem of robustified sequential decision rules construction (**?**).

2 Short Description of Data Models, Distortions and Results

Three types of data models were considered: sequences of independent identically distributed observations; Markov chains; time series with a trend. Simple and composite (in the Bayesian setting) hypotheses cases are studied.

The following distortion types were analyzed: "outliers" in the data; functional distortions of the likelihood presented by ε-neighborhoods; "contamination" of the prior probability density function. The incomplete data case is investigated for the model of time series with a trend.

Asymptotic expansions of performance characteristics (factual values of error probabilities, conditional expected sample sizes) are derived for the proposed families of generalized sequential decision rules, including the decision rules that are traditionally used. Within the proposed families the robustified sequential decision rules are constructed. Theoretical results are illustrated numerically.

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