A nonparametric graphical model for functional data with application to brain networks based on fMRI

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We introduce a nonparametric graphical model whose observations on vertices are functions. Many modern applications, such as electroencephalogram and functional magnetic resonance imaging (fMRI), produce data are of this type. The model is based on Additive Conditional Independence (ACI), a statistical relation that captures the spirit of conditional independence without resorting to multi-dimensional kernels. The random functions are assumed to reside in a Hilbert space. No distributional assumption is imposed on the random functions: instead, their statistical relations are characterized nonparametrically by a second Hilbert space, which is a reproducing kernel Hilbert space whose kernel is determined by the inner product of the first Hilbert space. A precision operator is then constructed based on the second space, which characterizes ACI, and hence also the graph. The resulting estimator is relatively easy to compute, requiring no iterative optimization or inversion of large matrices. We establish the consistency the convergence rate of the estimator. Through simulation studies we demonstrate that the estimator performs better than the functional Gaussian graphical model when the relations among vertices are nonlinear or heteroscedastic. The method is applied to an fMRI data set to construct brain networks for patients with attention-deficit/hyperactivity disorder.