A robust estimator for the mean direction of the von Mises-Fisher distribution

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The von Mises-Fisher (or Langevin) distribution is commonly used to describe the distribution of data on the (hyper)sphere. The distribution has two parameters: the mean direction $\boldsymbol{\mu}$ and the concentration parameter κ (with $0 \leq \kappa \leq \infty$), is unimodal (for $\kappa > 0$), and symmetrical about $\boldsymbol{\mu}$. Estimators for both parameters are readily available for some time (**???**). While robustness issues have been taken into consideration when estimating the concentration parameter (**???**), robustness has only recently been discussed when estimating the mean direction (**?**). This is mainly due to a common and persistent misconception that non-robustness does not constitute a serious problem when working with directional data (especially when talking about the mean direction, as the directional difference between any two points on the (hyper)sphere can only be 180° or π at the maximum).

In this paper the measurement of robustness of mean direction estimators is reconsidered. A new measure to assess the robustness based on the maximum bias is introduced. This measure provides the foundation to derive a definition of breakdown of an estimator similar in concept to the well-known finite sample breakdown point. Finally, a new estimator for the mean direction of the von Mises-Fisher distribution is proposed which is shown to deliver consistent estimates as well as being robust in terms of the newly introduced measures of robustness. Results of a simulation study indicate that the new estimator is more robust than the approach by **?** and much more robust than the standard ML estimator for certain contamination schemes.

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