Non-unbiased two-sample nonparametric tests. Numerical example

X. Yermolenko[1*]

[1] *Charles University in Prague, Department of Statistics, Sokolovská 83, CZ-186 75 Prague 8, Czech Republic; Yermolenko@karlin.mff.cuni.cz*
[*] *Presenting author*

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Many tests on vector or scalar parameters against two-sided alternatives are generally not finite-sample unbiased. They are unbiased only for symmetric distributions or under similar conditions. This was already noticed by **?**, **?** and generally analyzed by **?**, **?** and later by many others. While in univariate models the tests are unbiased against one-sample alternatives, such alternatives are not clearly characterized in the multivariate models.
We shall numerically illustrate this important problem on the Wilcoxon test against two-sample alternative of shift in location, applied to a skew logistic distribution and unequal sample sizes. Namely, let $X_1, \ldots, X_n$; $Y_1, \ldots, Y_m$ be two random samples according to absolutely continuous distributions $F(x)$ and $G(x)$ respectively.
In order to test the hypotheses $G(x) = F(x)$, against the alternative that F is the skew logistic distribution and $G(x) = F(x-\Delta), \Delta \neq 0$, we will consider the following two - sample Wilcoxon test:

$$\phi(X_1, \ldots, X_n, Y_1, \ldots, Y_m) = \begin{cases} 1, & \text{if } X_{(n)} < Y_{(1)} \text{ or } X_{(1)} > Y_{(m)}, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

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